Defeasible Reasoning Meets Epistemic Possibilities

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Abstract. The paper proposes a semantics to account for the role of epistemic possibilities in defeasible reasoning, especially for how epistemic possibilities block defeasible consequences, e.g. while that Tweety flies defeasibly follows from the premises that if Tweety is a bird, then normally Tweety flies and that Tweety is a bird, that Tweety flies no longer defeasibly follows once we add the additional premise that Tweety might not fly (e.g. for the reason that Tweety is injured). The proposed semantics, based on the semantics for defeasible reasoning (commonsense entailment) in Asher & Morreau (1991, 1995), builds on top of the update semantics for epistemic possibilities in Veltman (1996). We show that the semantics properly accounts for a variety of data concerning epistemic possibilities in defeasible reasoning. Moreover, a connection between negated defaults and epistemic possibilities naturally follows so that we can have a simpler and more intuitive account for the effect of negated defaults in defeasible reasoning.

Keywords. Defeasible Reasoning, Epistemic Possibilities, Update Semantics, Negated Defaults, Commonsense Entailment

1 Motivating Data

Not much has been said concerning the role of epistemic possibilities in defeasible reasoning (commonsense entailment). Nonetheless, epistemic possibilities play an important role in defeasible reasoning, especially in defeating defeasible consequences. For example, while (1c) defeasibly follows from (1a,b), neither (2d) nor (3d) follow from (2a,b,c) and (3a,b,c), respectively.

(1) (a) If Tweety is a bird, then normally Tweety flies. (b) Tweety is a bird. (c) / ∴ Tweety flies.

(2) (a) If Tweety is a bird, then normally Tweety flies. (b) Tweety is a bird. (c) Tweety does not fly. (d) */ ∴ Tweety flies.

(3) (a) If Tweety is a bird, then normally Tweety flies. (b) Tweety is a bird. (c) Tweety might not fly. (d) */ ∴ Tweety flies.

Given that defeasible consequences are what we 'expect' from a give set of premises, consider how (2c) defeats (2d) from Principle NC: there should be no contradictory expectations unless the premises are contradictory. Given the principle, (2c) defeats (2d) because expecting (2d) contradicts (2c). If we adopt this picture of how (2c) defeats (2d), we have an intuitive difficulty to account for how (3c) defeats (3d): in standard modal logic, expecting (3d) does not contradict (3c). So in a semantics that accounts for how epistemic possibilities defeat, we need to revise Principle NC.

2 Defeasible Update Semantics (DUS)

To account for how epistemic possibilities defeat, we extend the semantics for defeasible reasoning in Asher & Morreau (1991, 1995) in the fashion of update semantics on epistemic possibilities from Veltman (1996), and our formalism shall be called *defeasible update semantics (DUS)*.

2.1 Defeasible Implications in DUS

The language $L_>$ for DUS is the language for propositional modal logic with a binary connective '$>$' to represent the defeasible implication between formulas. Let $FL_\Diamond$ be the formulas defined in propositional modal logic with the set of sentential letters SL and the modal operator $\Diamond$. To avoid unnecessary complication, we avoid any occurrence of $\Diamond$ and $\rightarrow$ in the antecedent of a formula in the form of $\varphi > \psi$. So a formula φ is a $L_>$ formula, denoted by $FL_>$, iff $\varphi \in FL_\Diamond$ or $\varphi = \psi > \chi$ where $\psi \in FL_\Diamond$ but there is no occurrence of $\Diamond$ and $\rightarrow$ in ψ and $\chi \in FL_\Diamond$. Following Asher & Morreau (1991, 1995), we paraphrase $\varphi > \psi$ as *if φ, then normally ψ*. A frame for DUS is defined as follows.

DEFINITION 1. A DUS frame is a pair $F =< W, ^\otimes >$, where

1. $W = \{0,1\}^{SL}$, where SL is the set of sentential letters in $L_>$, and
2. $^\otimes : \wp(W)^{\wp(W)} \rightarrow \wp(W)^{\wp(W)}$, where $^\otimes$ satisfies the following constraints: for any $S \subseteq W$ and $\| \varphi \| \in \wp(W)^{\wp(W)}$,
   (a) **Facticity.** $^\otimes(\| \varphi \|)(S) = \| \varphi \| (^\otimes(\| \varphi \|)(S))$, and
   (b) **Inheritance.** For any information state S, $^\otimes(\| \varphi|)(S) = {}^\otimes(\| \varphi|)(W) \cap S$.

A DUS model M$=< W, ^\otimes, \| \,.\, \| >$ is a triple. The function $\| \,.\, \|: FL_> \rightarrow (\wp(W)^{\wp(W)})$ is the interpretation function for formulas in $L_>$ ($FL_>$) defined in a relational matter between *information states*, i.e. sets of possible worlds.

DEFINITION 2. **Basic Updates.**

1. $S \parallel p \parallel = \{w \in S : w(p) = 1\}$
2. $S \parallel \varphi \wedge \psi \parallel = S \parallel \varphi \parallel\parallel \psi \parallel$
3. $S \parallel \neg\varphi \parallel = S - S \parallel \varphi \parallel$
4. $S \parallel \varphi \vee \psi \parallel = S \parallel \varphi \parallel \cup S \parallel \psi \parallel$
5. $S \parallel \Diamond\varphi \parallel =$
   (a) S, if $S \parallel \varphi \parallel \neq \emptyset$, and
   (b) $\emptyset$, otherwise.
6. $S \parallel \varphi \rightarrow \psi \parallel =$
   (a) S, if $S \parallel \varphi \parallel\parallel \psi \parallel = S \parallel \varphi \parallel$,
   (b) $\emptyset$, otherwise.
7. $S \parallel \varphi > \psi \parallel =$
   (a) S, if $S^{\otimes}(\parallel \varphi \parallel) \parallel \psi \parallel = S^{\otimes}(\parallel \varphi \parallel)$,
   (b) $\emptyset$, otherwise.

Definition 2 differs from Veltman (1996) only on clause 7. Clause 7 makes formulas like $\varphi > \psi$ a test in the sense that $S^{\otimes}(\parallel \varphi \parallel)$ outputs the set of normal φ worlds in S and S passes the test of $\varphi > \psi$ just in case that the normal φ worlds in S are also ψ worlds. Logical consequences for DUS are defined as follows.

DEFINITION 3. **Logical Consequences.**

1. For a model M and an information state S, $S \models_M \varphi$ (S supports φ) iff $S \parallel \varphi \parallel_M = S$.
2. $\Gamma \models \varphi$ iff for any M and S, $S \parallel \varphi_1 \parallel_M ... \parallel \varphi_n \parallel_M \parallel \varphi \parallel_M = S \parallel \varphi_1 \parallel_M ... \parallel \varphi_n \parallel_M$, where $\Gamma = (\varphi_1, ..., \varphi_n)$ is a sequence.
3. $\models \varphi$ iff for any M and S, $S \models_M \varphi$

What shall be shown to be relevant to how epistemic possibilities defeat is the following fact, and Principle NC is replaced by Principle ES accordingly.

FACT 4. **Exclusive Support.** Two formulas φ and ψ are 'exclusively supported' iff for any model M and information state $S \neq \emptyset$, if $S \models_M \varphi$ (i.e. $S \parallel \varphi \parallel_M = S$) then, $S \not\models_M \psi$ (i.e. $S \parallel \psi \parallel_M \neq S$). φ and $\neg\varphi$, as well as for φ and $\Diamond\neg\varphi$, are exclusively supported.

- *Principle ES.* There should be not exclusively supported expectations unless the premises are contradictory.

2.2 Defeasible Consequences in DUS

Following the idea in Asher & Morreau (1991, 1995), we build the defeasible consequence relation $\approx\!\!|$ on top of the logical consequence $\models$: to see what an information state S 'defeasibly supports' after update with a set of premises Γ, denoted as $S[\Gamma]$, we check what is supported by some normalization of $S[\Gamma]$ with respect to Γ, denote as $S[\Gamma]_N$, in the sense that an $S[\Gamma]_N$ represents a way of getting rid of abnormal worlds in S with respect to Γ. We spell out the required notions as follows.

DEFINITION 5. **Normalization Functions.**

$\underline{N}(S, \| \varphi \|, \Gamma) =$

1. $S^{\otimes}(\| \varphi \|)$, if
   (a) $S^{\otimes}(\| \varphi \|) \neq \emptyset$, and
   (b) for any $\varphi_n \in \Gamma$, if $\Gamma \models \varphi_n$, then $S^{\otimes}(\| \varphi \|) \| \varphi_n \| = S^{\otimes}(\| \varphi \|)$,
2. S otherwise.

A normalization chain is defined on top of a set of normalization propositions: the normalization propositions of a set of formulas Γ, $P_\Gamma = \{\varphi : \varphi \in \wedge\Gamma \text{ and } \varphi > \psi \in \Gamma\}$, where $\varphi \in \wedge\Gamma$ iff (a) $\varphi \in \Gamma$, or (b) $\varphi = \psi \wedge \chi$ such that $\psi, \chi \in \wedge\Gamma$, or (c) there is a χ and a ψ such that $\chi \in \Gamma$ and that $\chi = \varphi \wedge \psi$ or $\chi = \psi \wedge \varphi$.

DEFINITION 6. **Normalization Chain.** Let $\nu : P_\Gamma \to N$ be an enumeration of a given P_Γ. A normalization chain for S with respect to Γ is defined with respect to an numeration ν on P_Γ, denoted by $S_{(\Gamma,\nu)}$, as follows.

1. ${S_{(\Gamma,\nu)}}^{0} = S[\Gamma]$ and ${\Lambda_{(\Gamma,\nu)}}^{0} = \emptyset$,
2. ${S_{(\Gamma,\nu)}}^{n+1} =$
   (a) $\underline{N}({S_{(\Gamma,\nu)}}^{n}, \| \varphi \|, \Gamma)$ and ${\Lambda_{(\Gamma,\nu)}}^{n+1} = {\Lambda_{(\Gamma,\nu)}}^{n} \cup \{\psi : \varphi > \psi \in \Gamma\}$, if $\nu(\varphi) = n + 1$ and for any $\psi \in {\Lambda_{(\Gamma,\nu)}}^{n}$, $\underline{N}({S_{(\Gamma,\nu)}}^{n}, \| \varphi \|, \Gamma) \| \psi \| = \underline{N}({S_{(\Gamma,\nu)}}^{n}, \| \varphi \|, \Gamma)$,
   (b) ${S_{(\Gamma,\nu)}}^{n}$ and ${\Lambda_{(\Gamma,\nu)}}^{n+1} = {\Lambda_{(\Gamma,\nu)}}^{n}$, otherwise, and
3. ${S_{(\Gamma,\nu)}}^{\omega} = \cap_{i \in N} {S_{(\Gamma,\nu)}}^{i}$, and ${\Lambda_{(\Gamma,\nu)}}^{\omega} = \cup_{i \in N} {\Lambda_{(\Gamma,\nu)}}^{i}$.

We say that ${S_{(\Gamma,\nu)}}^{\alpha}$ is a fix point of $S_{(\Gamma,\nu)}$, denoted as ${S_{(\Gamma,\nu)}}^{fix}$, iff for any $\beta > \alpha$, ${S_{(\Gamma,\nu)}}^{\alpha} = {S_{(\Gamma,\nu)}}^{\beta}$.

DEFINITION 7. **Reasons.** Assume Γ is a set of premises, ν is an enumeration on P_{Γ}, $M =< W, ^{\otimes}, \| . \| >$ is a model, S is an information state, $S[\Gamma] \neq \emptyset$, where $S[\Gamma] = S \parallel \varphi_1 \parallel ... \parallel \varphi_n \parallel$ if $\Gamma = (\varphi_1, ..., \varphi_n)$, and for every $\alpha \in P_{\Gamma}$, $S[\Gamma]^{\otimes}(\| \alpha \|) \neq \emptyset$. ${S_{(\Gamma,\nu)}}^{fix}$ is a reason from Γ, iff, for some n, ${S_{(\Gamma,\nu)}}^{n+1} = \underline{N}({S_{(\Gamma,\nu)}}^{n}, \| \varphi \|, \Gamma) = {S_{(\Gamma,\nu)}}^{n\otimes}(\| \varphi \|)$ under the condition that for any $\varphi_n \in \Gamma$, if $\Gamma \models \varphi_n$, then $S^{\otimes}(\| \varphi \|) \parallel \varphi_n \parallel = S^{\otimes}(\| \varphi \|)$.

DEFINITION 8. **Defeasible Consequences.** $\Gamma \approx\!\!\!|\; \varphi$ iff either $\Gamma \models \varphi$, or the following two requirements are satisfied: (a) *the existential requirement:* there is a reason ${S_{(\Gamma,\nu)}}^{fix}$ from Γ such that ${S_{(\Gamma,\nu)}}^{fix} \models \varphi$, and (b) *the universal requirement:* for every reason ${S_{(\Gamma,\nu)}}^{fix}$ from Γ, ${S_{(\Gamma,\nu)}}^{fix} \models \varphi$.

We say that a set of premises Γ is supportable, iff there is a model M and information state $S \neq \emptyset$ such that $S[\Gamma] \neq \emptyset$, and we say that a set of formulas Γ is further supportable iff $\Gamma^{\rightarrow}$ is supportable, where $\Gamma^{\rightarrow}(x) = \Gamma(x)$ iff $\Gamma(x) \neq \varphi > \psi$ for some φ and ψ, and $\Gamma^{\rightarrow}(x) = \varphi \rightarrow \varphi$ iff $\Gamma(x) = \varphi > \varphi$. Our notions of defeasible consequences can validate theorems validated in various systems of defeasible reasoning, e.g. Defeasible Modus Ponens, Defeasible Chaining, Nixon Diamond, etc. We list some of them as follows, and theorem 13 accounts for epistemic possibilities defeat in example (3).

THEOREM 9. *If* $\Gamma \models \varphi$, *then* $\Gamma \approx\!\!\!|\; \varphi$

THEOREM 10. **Defeasible Modus Ponens.** *If* $(\varphi > \psi, \varphi)$ *is further supportable,* $(\varphi > \psi, \varphi) \approx\!\!\!|\; \psi$.

THEOREM 11. **Defeasible Chaining.** *If* $(\varphi > \psi, \chi \rightarrow \varphi, \chi)$ *is further supportable, then* $(\varphi > \psi, \chi \rightarrow \varphi, \chi) \approx\!\!\!|\; \psi$.

THEOREM 12. *If* $(\varphi > \psi, \varphi, \neg\psi)$ *is supportable,* $\varphi > \psi, \varphi, \neg\psi \not\approx\!\!\!|\; \psi$

THEOREM 13. *If* $(\varphi > \psi, \varphi, \Diamond\neg\psi)$ *is supportable,* $(\varphi > \psi, \varphi, \Diamond\neg\psi) \not\approx\!\!\!|\; \psi$

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